Abstract
Lag-lead compensators are well known in automatic control engineering. They have 3 or 4 parameters to be adjusted (tuned) for proper operation depending on the compensator order. The frequency response of the control system or the root locus plot are traditionally used to tune the compensator in a lengthy procedure.

A selected simple pole plus double integrator process is controlled using a first-order lag-lead compensator (through simulation). The lag-lead compensator is tuned by minimizing the sum of square of errors of phase margin plus gain margin of the compensated system using MATLAB. Two functional constrains are used to guarantee the stability of the lag-lead compensated control system. The result was maintaining the gain margin and phase margin of the control system exactly at the desired values. The maximum percentage overshoot and the settling time depends on the desired gain margin. The steady-state error of closed-loop control system using the first-order lag-lead compensator with the studied process is zero. The results are compared with that in a published work.

Keywords
First-order lag-lead compensator - Compensator Tuning - Simple pole plus double integrator process - Control system performance.

I. Introduction
Lag-lead compensators can improve the performance of linear control systems through the proper tuning of the compensator parameters. There are two schools in designing lag-lead compensators. One of them uses the frequency response specifications of the compensated control system. The other uses its time response specifications. In the present work we follow the research school using the frequency response specifications. Still the subject is interested to automatic control researchers. James, Frederick and Taylor (1987) discussed the application of expert system technique to the design of lead-lag compensators for linear SISO systems [1]. Loh, Cai and Tan (2004) studied the auto-tuning of phase lead-lag compensators using the frequency response of the plant using relays with hysteresis [2]. Chang (2004) used phase-lag and phase-lead compensators to control servo control systems [3].


Nagshi, Rahmani, Vahidi and Hosseinion (2013) introduced a combination of static VAR and a lag-lead controller to enhance the dynamic stability of power systems. They achieved better dynamic performance and reduced steady-state error [14]. Mahmoud (2013) described short steps to design a phase-lead compensator for the mass-spring-damper system to achieve the desired level of its phase margin. He used a first-order phase lead compensator [15]. Hassaan, Al-Gamil and Lashin (2013) used the sum of absolute error criterion to tune a lag-lead compensator used with a first-order process plus an integrator process. Their tuned compensator could reduce the maximum overshoot from 67.3 % to 2.44 % and the settling time from 12 to 0.65 seconds [16]. Bansal and Dewan (2014) investigated the application of PID control a series compensator to stabilize a gimbal system. They were in favor of using a PID controller against using a simple compensator [17]. Sedaghati et al. (2014) used a 2-stage lead-lag compensator compared with another types of controllers to provide the damping required to stabilize power systems [18]. Morishita, Suzuki and Iwamoto (2014) described obtaining robustness of a power system stabilizer of a lead-lag compensator using the $H_{\infty}$ control. They optimized the parameters of the compensator using a particle swarm optimization with evolution function considering the closed-loop $H_{\infty}$ norm and a desired response [19].

II. Analysis
Process
The process used in the present study is one of the difficult
processes consisting of one simple pole and two integrators. This process is unstable either if used in an open-loop fashion, or if used in a unity feedback loop. It has been used by Ogata and has the transfer function [14]:

\[ G_p(s) = \frac{1}{s^2(s + 5)} \]  

The process has the time response to a unit step input shown in Fig.1.

\[ M(s) = \frac{b_0 s + b_1}{a_0 s^5 + a_1 s^4 + a_2 s^3 + a_3 s^2 + a_4 s + a_5} \]  

where:

- \( b_0 = K_c T_z \)
- \( b_1 = K_c \)
- \( a_0 = T_z T_p \)
- \( a_1 = T_p + T_z (1 + 5 T_p) \)
- \( a_2 = 1 + 5 T_z + 5 T_p \)
- \( a_3 = 5 \)
- \( a_4 = K_c T_z \)
- \( a_5 = K_c \)

### Frequency-based and Time-based Specifications:
- Eq.3 is used to assign the system gain and phase margins of the control system using the MATLAB command “margin”.
- Eq.4 is used to assign the maximum percentage overshoot and settling time using the MATLAB command “stepinfo”.

### III. Tuning of The First-Order Lag-Lead Compensator
The sum of square of error (ISE) is used as an objective function, \( F \) of the optimization process. The error function here is assumed as the sum of the square of difference between the system phase margin and its desired value and difference between the system gain margin and its desired value. That is:

\[ F = \left[ PM - PM_{des} \right]^2 + \left[ GM - GM_{des} \right]^2 \]  

Where:

- \( PM = \) system phase margin.
- \( PM_{des} = \) desired phase margin.
- \( GM = \) system gain margin.
- \( GM_{des} = \) desired gain margin.

The compensator parameters have to assigned such that the closed-loop control system is stable. This is achieved through two functional constraints defined from the Routh-Hurwitz criterion of the system. That is:

\[ c_1 = a_0 a_3 - a_2 a_1 \]  

And:

\[ c_2 = a_2 a_3 - a_1 a_3 \]  

\[ c_3 = a_0 a_5 - a_2 a_1 \]  

Where:

\[ a_1 = \frac{(a_0 a_2 - a_3 a_1)}{a_1} \]
\[ a_2 = \frac{(a_0 a_3 - a_2 a_2)}{a_1} \]
\[ a_3 = \frac{(a_1 a_2 - a_2 a_3)}{a_1} \]

### Parameters Limits:
- A lower limit of 0.001 is set for the compensator parameters: \( T_z, T_p \) and \( K_c \).
- An upper limit of 100 is set for the three compensator parameters.

### IV. Tuning Results
The MATLAB command “fmincon” is used to minimize the optimization objective function given by Eq.5 subjected to the functional inequality constraints given by Eqs. 6, 7 and 8 to provide the first-order lag-lead compensator parameters subjected to the limits mentioned in section 6. The results depends on the desired gain margin of the control system. Tables 1 and 2 give the first-order compensator parameters and the control system specifications for a desired phase margin of 50° and a gain margin.
in the range: 6 – 30 dB.

Table 1: Compensator parameters.

<table>
<thead>
<tr>
<th>GM_{desired} (dB)</th>
<th>T_z (s)</th>
<th>T_p (s)</th>
<th>K_c</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3.5887</td>
<td>0.1159</td>
<td>2.6574</td>
</tr>
<tr>
<td>8</td>
<td>2.9410</td>
<td>0.1417</td>
<td>2.2661</td>
</tr>
<tr>
<td>10</td>
<td>2.6127</td>
<td>0.1284</td>
<td>2.1486</td>
</tr>
<tr>
<td>12</td>
<td>2.4220</td>
<td>0.0937</td>
<td>2.1159</td>
</tr>
<tr>
<td>14</td>
<td>2.3084</td>
<td>0.0788</td>
<td>2.1517</td>
</tr>
<tr>
<td>16</td>
<td>2.2267</td>
<td>0.0731</td>
<td>2.0630</td>
</tr>
<tr>
<td>18</td>
<td>2.2098</td>
<td>0.0722</td>
<td>1.8388</td>
</tr>
<tr>
<td>20</td>
<td>2.2646</td>
<td>0.0638</td>
<td>1.8706</td>
</tr>
<tr>
<td>22</td>
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<td>0.0634</td>
<td>1.6744</td>
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<tr>
<td>24</td>
<td>2.2981</td>
<td>0.0535</td>
<td>1.8382</td>
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<tr>
<td>26</td>
<td>2.1949</td>
<td>0.0506</td>
<td>1.7719</td>
</tr>
<tr>
<td>28</td>
<td>2.2138</td>
<td>0.0497</td>
<td>1.6461</td>
</tr>
<tr>
<td>30</td>
<td>2.2635</td>
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</table>

Table 2: Control system specifications

<table>
<thead>
<tr>
<th>GM_{desired} (dB)</th>
<th>GM (dB)</th>
<th>PM (deg)</th>
<th>OS (%)</th>
<th>T_s (s)</th>
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<tr>
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<td>6.536</td>
<td>49.81</td>
<td>24.0534</td>
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<td>8</td>
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<td>50</td>
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<td>50</td>
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<td>30</td>
<td>50</td>
<td>29.383</td>
<td>7.0469</td>
</tr>
</tbody>
</table>

The time response of the compensated system to a unit step input for a 50° phase margin and a 16 dB gain margin is shown in Fig. 2.

Ogata used the frequency response analysis to assign the first-order compensator parameters used to control this unstable process for a 50° desired phase margin and a gain margin > 10 dB [20]. His parameters are:
- \( T_z = 3.9 \) s (compared with 2.2267 s in the present study).
- \( T_p = 0.2557 \) s (compared with 0.0788 s in the present study).
- \( K_c = 1.3043 \) (compared with 2.1517 in the present study).

Those parameters resulted in a stable control system having the time response to a unit step input shown in Fig. 3.

*V. Comparison With Ogata Results*

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- \( K_c = 1.3043 \) (compared with 2.1517 in the present study).

Those parameters resulted in a stable control system having the time response to a unit step input shown in Fig. 3.

The control system using Ogata parameters has the performance characteristics:
- Phase margin: 49.9965 degrees
- Gain margin: 16 dB
- Maximum percentage overshoot: 25.82 % (compared with 28.497 % in the present study).
- Settling time: 7.03 s (compared with 5.94 s in the present study).

*VI. Conclusion*

- The suggested tuning technique of first-order lag-lead compensators used with an unstable process is simple and efficient.
- Frequency-based control system specifications are used in the tuning technique in conjunction with the MATLAB optimization toolbox.
- Using the proposed tuning technique, it was possible to adjust the settling time to from 4.5 to 7 seconds depending on the desired gain margin.
- The maximum percentage overshoot was in the range 24-29 % depending on the desired gain margin.
- Using the time-based specifications reveals better tuning of the compensator since the time-based specifications will be under control.
- The steady-state error associated with a unit step input was zero.
- The optimization tuning approach used in this work is simple, straightforward, and provides the compensator parameters in a very small time using MATLAB.
References


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