A Novel Notch Compensator used with a Highly Oscillating Second-Order Process

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Abstract
Compensators are used in place of classical PID controllers for possible achievement of better performance of the closed loop control system. Highly oscillating processes require more effort in selecting a proper controller or compensator. In this paper a novel compensator based on the notch filter is proposed and applied to control a process having 85% overshoot. The compensator proposed used a gain element in series with a notch filter to control the steady-state characteristics of the closed-loop control system. The advantage of the proposed compensator is its simple tuning without need to an advanced optimization technique. It was possible with the notch compensator with gain to satisfy a system performance without any overshoot with a settling time as low as 0.03 seconds and steady-state error as low as 0.0066 for a unit step input.

Keywords

I. Introduction
Feedforward compensators find wide application in both linear and nonlinear dynamic systems. The design of classical compensators such as lag, lead, lag-lead, PID and pre-filter are investigated in automatic control textbooks [1-5]. Giusto and Paganini (1999) considered the design of feedforward compensator for robust H∞ or H2 performance under structured uncertainty [6]. Ro, Shim and Jeong (2000) developed a PD control scheme with a nonlinear friction estimate algorithm for a ball-screw-driven slide system. Their approach resulted in a system performance with steady-state error under 1.5% [7]. Messner and McNob (2001) presented a method for the design of compensators for linear time-invariant dual-input / single-output system. They reduced the problem to 2-single input / single output design problems [8]. Shi, Dimirovski, Zhang and Stankovski (2002) introduced a theoretical approach generating a new design method. They designed normal dynamic compensators such that the closed-loop control system is asymptotically stable and has no impulse effect [9]. Choi, Son, Han and Choi (2003) presented a micro-positioning mechanism with dual servo system. They used a lead compensator for the control of the system [10]. Gessing (2004) used Smith compensator connected in parallel to the plant. The proposed approach is to simplify the control system design and improve the control accuracy [11]. Wu and Duan (2005) addressed the design of a type of dynamical compressors for matrix second order linear systems in the matrix framework using a complete parametric approach [12]. Leva and Bascetta (2006) presented a methodology to design the feedforward path of 2DOF regulators for optimum set point tracking using a nonparametric model of the control loop identified online[13]. Panda and Padhy (2007) applied the genetic algorithm optimization technique to design a thyristor controlled series compensator to enhance the power system stability using a lead-lag and a PID compensators [14]. Rico and Camacho (2008) presented a review of the main dead-time compensators designed to improve the closed loop characteristics and to control unstable systems [15]. Shu’aibn and Adamu (2009) presented the design of a phase-lead compensator (PD-controller) of a magnetic levitation system using the root-locus method and the MATLAB control toolbox [16]. Roy, Wan, Saberi and Malek (2010) presented a methodology for designing low gain linear time-invariant compensators for semi-global stabilization using a pre-compensator plus a static output feedback [17]. Zanasi, Cuoghi and Ntogramatzidis (2011) presented the dynamic structure and the control properties of a new form of lead-lag compensator with complex zeros and poles [18]. Mori (2012) investigated the 2-stage compensator design of linear systems. He derives various types of the 2-stage compensator design with partial feedbacks [19]. Espi (2013) presented an optimal capacitance design for the DC link of a back-to-back converter for wind turbine power generation. He used PI-compensator for the voltage regulation [20]. To suppress the high oscillations of second-order-like processes, Galal Hassaan (2013, 2014) of Cairo University proposed different schemes: using a PIDF controller [21], using a feedback first-order compensator [22] and using a PD-PI controller [23]. The proposed notch compensator is simple in design and construction than the classical lag-lead compensator. It is possible using this compensator to control both steady-state and dynamic characteristics of the control system.

II. Analysis
Process:
The process is a second order process having the transfer function,

\[ G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \]  

Where:
- \( \omega_n \) = process natural frequency = 10 rad/s
- \( \xi \) = process damping ratio = 0.05

The time response of this process in a unit feedback loop without compensation to a unit step input is shown in Fig.1 as generated by MATLAB:
The performance of the process is measured by its maximum percentage overshoot and its settling time. It has a maximum overshoot of 85.45 % and about 6 seconds settling time.

The Notch-filter
A notch filter has a second order transfer function, $G_f(s)$ of the specific form [24,25]:

$$G_f(s) = \frac{s^2 + b_1}{s^2 + a_1s + b_1} \quad (2)$$

It has the 2 parameters $b_1$ and $a_1$, which are function of the values of the resistance and capacitance of the resistor and capacitor components encountered in the filter electronic circuit [25].

A Notch Compensator
A notch compensator consists of the classical notch filter of Eq.2 and a series variable gain element of gain $K$ to control the steady-state characteristics of the closed-loop control system incorporating the feedforward compensator and the process. The compensator has the transfer function, $G_c(s)$:

$$G_c(s) = \frac{K(s^2 + b_1)}{s^2 + a_1s + b_1} \quad (3)$$

The compensator has 3-parameters:
- $K$: Compensator gain
- $b_1$: Notch filter parameter
- $a_1$: Notch filter parameter

Control System Transfer Function
Assuming that the control system is a unit feedback one, its transfer function with $G(s)$ of Eq.3 and $G_p(s)$ of Eq.1 is:

$$M(s) = \frac{\beta_0 s^2 + \beta_1}{s^4 + \alpha_1s^3 + \alpha_2s^2 + \alpha_3s + \alpha_4} \quad (4)$$

where:
- $\beta_0 = K \omega_n^2$
- $\beta_1 = K\omega_n^2 b_1$
- $\alpha_1 = 2\zeta\omega_n + a_1$
- $\alpha_2 = \omega_n^2 (1+K) + 2\zeta\omega_n a_1$
- $\alpha_3 = \omega_n^2 a_1 + 2\omega_n b_1$
- $\alpha_4 = \omega_n^2 b_1 (1+K)$

III. Stability of The Closed-Loop Control System
The compensator parameters have to be determined such that the closed-loop control system is stable. Since the closed-loop system is a fourth order one, it is possible to be unstable. Therefore, the compensator parameters have to match the stability conditions of the control system. Using the characteristic equation which is the denominator of Eq.4 and the Routh-Hurwitz stability criterion [26], the stability conditions are:

1. Condition 1: $\alpha_1 \alpha_3 > \alpha_4$
   Written in the form:
   $$f_1 = \alpha_1 \alpha_3 - \alpha_4 > 0 \quad (5)$$

2. Condition 2: $\gamma_1 \alpha_1 > \alpha_1 \alpha_4$
   Written in the form:
   $$f_2 = \gamma_1 \alpha_1 - \alpha_1 \alpha_4 > 0 \quad (6)$$

where:
- $\gamma_1 = \alpha_2 - \alpha_3 / \alpha_1$

Eqs.5 and 6 plays an important role in tuning the notch-compensator. For example let: $K = 150$ and $a_1 = 200$. The variation of $f_1$ with the third compensator parameter $b_1$ is shown in Fig.2.

Fig.2 shows that any value of $b_1$ in the set range: $0 \leq b_1 \leq 200$ will produce a stable system which has to be confirmed by the second function $f_2$. The variation of $f_2$ with the third compensator parameter $b_1$ is shown in Fig.3.

Fig.3 depicts that the system is unstable for $b_1 > 100$. Therefore, any selected value for $b_1$ associated with $K=100$ and $a_1=200$ has
to be < 100. Through zooming the plot around the 100 value, the
critical value of $b_1$ is 100.666.

IV. System Step Response and Performance
A unit step response is generated by MATLAB using the numerator
and denominator of Eq. 4 providing the system time response $c(t)$
as function of time for a set of compensator parameters.
The characteristics of the compensated control system quantifying
its performance are:
- Steady-state response, $c_{ss}$:
  Using Eq.3, the system steady-state response for a unit step input
  is:
  $$c_{ss} = \frac{K}{1+K} \quad (7)$$
- Steady-state error, $e_{ss}$:
  Using Eq.3, the system steady-state error for a unit step input
  is:
  $$e_{ss} = \frac{1}{1+K} \quad (8)$$
- Maximum percentage overshoot, $OS_{max}$:
  Using the time response of the control system to a unit step input,
  the maximum percentage overshoot is:
  $$OS_{max} = 100 \left( \frac{c_{max} - c_{ss}}{c_{ss}} \right) \quad (9)$$
  Where: $c_{max}$ = maximum time response to a step
  input.
  $c_{ss}$ = steady-state response of the control system to the
  unit step input
  - Settling time, $T_s$:
  The time response of the system enters a band of ±5 % of the
  steady-state response and remains inside this band.

V. Notch Compensator Tuning
The compensator proposed in this work is tuned manually without
need to sophisticated optimization techniques. The procedure is
as follows:
- Assign the compensator gain $K$ according to the desired
  steady-state error using Eq.7.
- Assign a value for $a_1$.
- Using MATLAB, plot $f_2$ against $b_1$ using Eq.6 (as in Fig.3).
  This step determines the maximum value of $b_1$ for a stable
  control system.
- Discritize the range 0 to $b_{1_{max}}$ (say every 5).
- For each set of values for $K$, $a_1$, $b_1$ extract the overshoot and
  settling time using the command “stepinfo”.
- Repeat for different values of $a_1$.
- Select the appropriate combination of the compensator
  parameters satisfying the desired performance of the closed
  loop control system.

VI. Notch Compensator Application
The effectiveness of this new compensator is examined through
the highly oscillating second order process whose step response is
shown in Fig. 1. The step time response of the closed loop system
 incorporating the proposed compensator and the process is shown
in Figs.4, 5 and 6.
Suppose that using Fig.6, the performance of the closed loop control system corresponding to the compensator parameters: $K = 150$, $b_1 = 100$ and $a_1 = 200$ is satisfactory. Using MATLAB, the unit step response of the system using those parameters is shown in Fig.7.

![Fig.7](image-url) System time response for $K = 150$, $b_1 = 100$ and $a_1 = 200$.

The closed-loop control system using the quoted compensator parameters has the performance measures:

- Steady-state error: 0.0066
- Maximum percentage overshoot: 0.549%
- Settling time: 0.0287 s

**V. Conclusions**

- Highly oscillating processes represent a big industrial problem due to its side effects on the final product of some industries.
- A new notch filter-based forward compensator is proposed.
- An adjustable gain element is used in series with the notch filter to control the steady-state characteristics of the control system.
- A manual tuning approach is used.
- It was possible using the proposed compensator to produce a zero overshoot control system when using a process of 85% maximum overshoot.
- The highly oscillating process had about 6 seconds settling time. Using the proposed compensator it was possible to reduce the settling time to only 0.03 second.

**References**


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