On Tuning a Novel Feedforward 2/2 Second-Order Compensator to Control a Very Slow Second-Order-Like Process

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Abstract
In this paper a novel 2/2 feedforward compensator is proposed and applied to control a second-order-like very slow process having 150 seconds settling time. The proposed compensator is capable of controlling the steady-state characteristics of the closed-loop control system and its dynamic characteristics. The MATLAB command “fsolve” is used to solve a set of nonlinear equations leading to tuning the proposed compensator. The resulting control system dynamics using the proposed compensator are completely satisfactory. The time response of the control system to a step input has no overshoot, the settling time is reduced to only 0.2 seconds and the steady-state error is as low as 0.01 for a unit step input.

Keywords
Very slow processes - 2/2 second-order compensators - Compensator tuning - Control system performance.

I. Introduction
Feedforward compensators find wide application in both linear and nonlinear dynamic systems. The design of classical compensators such as lag, lead, lag-lead, PID and pre-filter are investigated in automatic control textbooks [1-5]. Randal and Sinencio (1985) discussed the application of the operational transconductance amplifier (OTA) in first-order and second-order filters structures [6]. Angulo and Snencio (1994) studied the low pass, high pass and band pass second-order filters using the multiple output nonlinearized operational transconductance amplifiers [7]. Edwards and Cauwenberghs (1998) described the synthesis of a second-order log-domain band pass filter and showed experimental results [8]. Ilehenko, Savchenkov, Handley and Maleki (2003) demonstrated an approach for fabricating a photonic filter with second-order response function consisting of two Germania-doped silica microtoroidal cascaded in series [9]. Wilamowski and Gottiparthy (2005) described an educational MATLAB tool simplifying the process of analog filter design for several types of filters [10]. Menekay, Tarcan and Kuntman (2007) designed a current-mode square-root circuit with reduced short channel effect proposing a second-order low pass filter [11]. Zambon and Lehtonen (2008) presented a sustain-pedal effect simulation algorithm for piano synthesis using parallel second-order filters [12]. Crump (2010) studied the first-order, second-order and biquadratic filters examining their responses to simplify the filter design task for design engineers [13]. Ibarra (2011) offered the basics on active filter design by introducing the Butterworth approach as well as some practical examples [14]. Pandey, Singh, Kumar, Dubey and Tyagi (2012) presented a current mode second-order filter employing single current differentiating transconductance amplifiers using current mode approach providing high pass, band pass and low pass responses [15]. Vishal, Saurabh, Singh and Chauhan (2012) studied the implementation of second-order low pass, high pass and band pass filters by using the current-controlled current differentiating buffered amplifier [16]. Hassaan, Al-Gamil and Lashin (2013) used a lag-lead second-order compensator to control a first-order plus an integrator process. They tuned the compensator through minimizing the sum of square of error objective function and could reduce the maximum overshoot to 2.43 % and the settling time to 0.65 s [17]. Hassaan (2014) suggested specific controllers and compensators to deal with second-order-like processes having very slow step-time response [19-21].

II. Analysis

Process

The process is a second-order process consisting of two simple-poles and a process gain. It has the transfer function:

$$G_p(s) = \frac{K_p}{(1 + T_1 s)(1 + T_2 s)}$$

Where:

$$K_p = 1$$,
$$T_1 = 0.1$$ and
$$T_2 = 50$$ s

The process transfer function of Eq.2 can be written in the standard form:

$$G_p(s) = K_p \omega_{np}^2 / (s^2 + 2\zeta_p \omega_{np} s + \omega_{np}^2) \quad (1)$$

Where:

$$\omega_{np} = 1 / \sqrt{(T_1 T_2)} = 0.4472 \text{ rad/s}$$

and

$$\zeta_p = 0.5(T_1 + T_2) / \sqrt{(T_1 T_2)} = 11.203$$

The process has the time response to a unit step input shown in Fig.1.

Fig.1: Step response of the uncompensated process.
The process has a zero maximum overshoot and a 150 seconds settling time (indicating the very slow time response of the process).

The Proposed Compensator

The proposed compensator is a 2/2 feedforward second-order one having a transfer function, $G_c(s)$ given by:

$$G_c(s) = \frac{K_c(s^2 + 2\zeta_c\omega_{nc1}s + \omega_{nc1}^2)}{(s^2 + 2\zeta_c\omega_{nc2}s + \omega_{nc2}^2)}$$  (2)

It has the 5 parameters:
- Gain, $K_c$.
- Natural frequencies, $\omega_{nc1}$ and $\omega_{nc2}$
- Damping ratios, $\zeta_{c1}$ and $\zeta_{c2}$

The five parameters are function of the values of the resistance and capacitance of the resistor and capacitor components encountered in the filter electronic circuit.

The configuration of this compensator as defined by Eq.2 has the advantages:
- Easy cancellation of the undesired process poles.
- Easy replacement of the bad poles by better ones.
- Flexibility of setting the compensator poles and zeros through using potentiometers.

The quadratic zero of the compensator is used to cancel the bad quadratic pole of the process resulting in its very slow time response. That is:

$$\zeta_{c1} = \zeta_p$$  (3)

and $\omega_{nc1} = \omega_{np}$

Control System Transfer Function

Assuming that the control system is a unit feedback one, its transfer function with $G_c(s)$ of Eq.2 and $G_p(s)$ of Eq.1 considering the conditions in Eq.3 is:

$$M(s) = \frac{b_0}{(s^2 + a_1s + a_2)}$$  (4)

where:

$$b_0 = K_\omega_{np}^2$$

$$a_1 = 2\zeta_c\omega_{nc2}$$

$$a_2 = \omega_{nc2}^2 + K_\omega_{np}^2$$

System Step Response and Performance:

A unit step response is generated by MATLAB using the numerator and denominator of Eq. 4 providing the system response $c(t)$ as function of time for a set of compensator parameters.

The characteristics of the compensated control system quantifying its performance are:
- Steady-state error, $e_{ss}$:
  - Using Eq.3, the steady-state error for a unit step input is:
    $$e_{ss} = 1 / \{1 + (K_\omega_{np}^2 / \omega_{nc2}^2 ) \}$$  (5)
  - Maximum percentage overshoot, $OS_{max}$:
    - Using the time response of the control system to a unit step input, the maximum percentage overshoot is:

$$OS_{max} = 100 \left( \frac{c_{max} - c_{ss}}{c_{ss}} \right)$$  (6)

Where: $c_{max}$ = maximum time response to a step input.

$\omega_{nc3}$ = steady state response of the control system to the unit step input.

- Settling time, $T_s$:
  - The time response of the system enters a band of $\pm 5\%$ of the steady-state response and remains inside this band.

III. Compensator Tuning

The compensator proposed in this work is tuned without need to a sophisticated optimization technique. The procedure is as follows:
- A desired steady-error $e_{ssdes}$, maximum percentage overshoot $S_{des}$ and settling time $T_{sdes}$ are assigned as a measure for the performance of the compensated control system.
- The following three nonlinear equations are formulated:

$$e_{ss} - e_{ssdes} = 0$$  (7)

$$OS - OS_{des} = 0$$  (8)

$$T_s - T_{sdes} = 0$$  (9)

- Eqs.7-9 are functions of the compensator parameters $K$, $\zeta_{c2}$ and $\omega_{nc2}$.
- The 3 equations are solved using the MATLAB command “fsolve” revealing the compensator parameters [22].

Tuning Results

Eqs.7-9 are solved using MATLAB for a desired control system performance defined by:
- Steady-state error: $e_{ssdes} = 0.01$
- Maximum percentage overshoot: $S_{des} = 0$
- Settling time: $T_{sdes} = 0.2$ s

Resulting in the compensator parameters:

$$K = 2386.2$$

$$\zeta_{c2} = 9.5$$

$$\omega_{nc2} = 2.2 \text{ rad/s}$$

The time response of the compensated system to a unit step input is shown in Fig.2.

![Fig. 2: Step response of the second-order compensated process.](image-url)
IV. Conclusions

- The suggested tuning technique of the second-order compensator is superior in handling a difficult processes such as the very slow second-order-like process.
- Using the proposed tuning technique, it was possible to reduce the settling time from 150 seconds to only 0.22 seconds.
- The compensator succeeded in producing a time response without any overshoot and/or undershoot.
- It was easy to control the steady-state characteristics of the closed-loop control system.
- A new tuning technique was presented without need to the application of any optimization technique.
- The formulated nonlinear equations were easily solved using one MATLAB command.

References


Authore Profile

- Emeritus Professor of Automatic Control and System Dynamics, Faculty of Engineering, Cairo University, Giza, EGYPT.
- Research in Automatic Control, Mechanical Vibrations and Mechanism Synthesis.
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