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Research on Signal Spectrum of Multirate Processing

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Abstract

The paper studies the influence of extraction and interpolation of multirate signal processing to signal spectrum, theoretical derivation of the signal spectrum after integer times extraction and integer times zeros values interpolation is done by taking advantage of Fourier transform, the regular pattern of signal spectrum after extraction and interpolation as well as the method of avoiding spectrum aliasing is obtained. The signal spectrum after integer times extraction and integer times zeros values interpolation is analyzed through simulation experiments to verify the correctness of theoretical derivation, some problems of multirate signal processing are pointed out to be aware of at last.

Keywords

Multirate Signal Processing; Extraction; Interpolation; Spectrum; FT

I. Introduction

The rapid development of digital signal processing technology leading to the increasing work of signal processing transmission and storage[1]. In order to reduce the computational workload and save storage space, different sampling rates and conversion are usually needed in one signal processing system. In this demand, multirate digital signal processing technology has been generated and developed since 1970s[2,3].

Multirate digital signal processing could change the sampling rate of digital signals through decimation and interpolation. Extraction is the conversion of reducing the signal sampling rate; while interpolation is the conversion of increasing the signal sampling rate. Multirate conversion has been introduced in a lot of material, but the research of spectral analysis in multirate conversion is not thoroughly. The spectrum of the signal after integer times extraction and integer times interpolation is deserved with Fourier Transform[4].

II. INTEGER TIMES EXTRACTION

Known continuous signal x(t), after equal intervals sampling with the rate of f1 = 1/T1 (T1 is the sampling interval) we get x(n). Sequence y(n)is the result of M times extraction of sequence x(n), which sampling rate is f2=1/T2.

$$y(n) = x(Mn) \quad (1)$$

According to the definition of the Fourier transform,

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x(n)e^{-jwn} \quad (2)$$

According to the nature of the Fourier transform,

$$Y(e^{jw}) = \sum_{n=-\infty}^{\infty} y(n)e^{-jwn} = \sum_{n=-\infty}^{\infty} x(Mn)e^{-jwn} \quad (3)$$

Let m=M*n, then m is an integer multiple of M.

$$\frac{1}{M}\sum_{K=0}^{M-1} e^{j\frac{2\pi}{M}km} = \frac{1-e^{j2\pi m}}{1-e^{j\frac{2\pi}{M}m}} = \begin{cases} 1; \text{ m/M} = \text{integer} \\ 0; \text{ m/M} \neq \text{integer} \end{cases} \quad (4)$$

According to (3) and (4),

$$Y(e^{jw}) = \sum_{m=-\infty}^{\infty} x(m)\left[\frac{1}{M}\sum_{K=0}^{M-1} e^{j\frac{2\pi}{M}km}\right] e^{-j\frac{w}{M}m} = \frac{1}{M}\sum_{K=0}^{M-1} X(e^{j(w-2\pi k)/M}) \quad (5)$$

It could be get from (5) that the spectrum of M-extraction signal is the result of the superposition and expand of the original signal spectrum.

If the spectrum of the input signal is greater thanπ/M, spectrum aliasing will generate, because the sample rate of the decimated signal could not less than the Nyquist sampling rate. Therefore, a low pass filter is needed prior to extraction, which cutoff frequency isπ/ M[5].

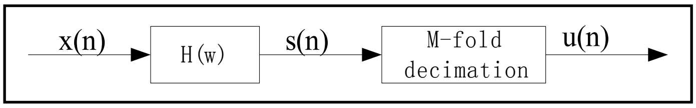

Fig. 1 : M-extractor Structure

The frequency response of the low pass filter:

$$H(e^{jw}) = \begin{cases} 1; & |W| < \frac{\pi}{M} \\ 0; & else \end{cases} \quad (6)$$

The spectrum of y(n) is:

$$Y(e^{jw}) = \frac{1}{M} X(e^{j\frac{w}{M}}) \quad |w| < \frac{\pi}{M} \quad (7)$$

III. Integer Times Interpolation

Sequence u(n) is the result of L times interpolation of sequence x(n).

$$u(n) = x\left(\frac{n}{L}\right) \quad (8)$$

According to the nature of the Fourier transform, we get the spectrum of u(n).

$$U(e^{jw}) = \sum_{n=-\infty}^{\infty} u(n)e^{jwn} = \sum_{n=-\infty}^{\infty} x(\frac{n}{L})e^{jwn} = \sum_{m=-\infty}^{\infty} x(m)e^{jwLm} = X(e^{jwL}) \quad (9)$$

It could be got from (9) that the spectrum cycle of L-interpolation signal is 1/L times of original one. Therefore, in the 2πrange of digital frequency axis will produce a mirror. In order to eliminate the mirror, a low pass filter is needed after interpolation, which cutoff frequency isπ/ L.

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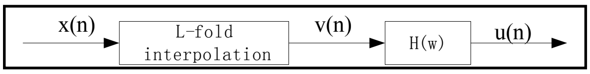

Fig. 2: L-interpolator Structure

The frequency response of the low pass filter:

$$H(e^{jw}) = \begin{cases} 1; |W| < \dfrac{\pi}{L} \\ 0; \text{else} \end{cases} \qquad (10)$$

The spectrum of u(n) is:

$$U(e^{jw}) = X(e^{jwL}) \quad |w| < \frac{\pi}{L} \qquad (11)$$

IV. Simulation Analysis

Known signal $x(t)=2\cos(2\pi f1t)+3\cos(2\pi f2t)$, f1=15Hz, f2=20Hz. Sequence x(n) is the sampling result of x(t) with the sampling rate of fs=100Hz. The following simulation is done to verify the influence of decimation and interpolation to signal spectrum.

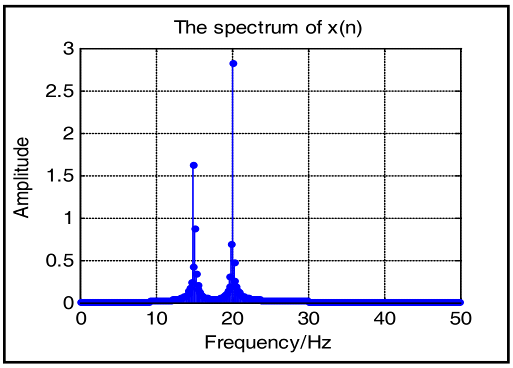

Fig. 3 : The spectrum of y(n)

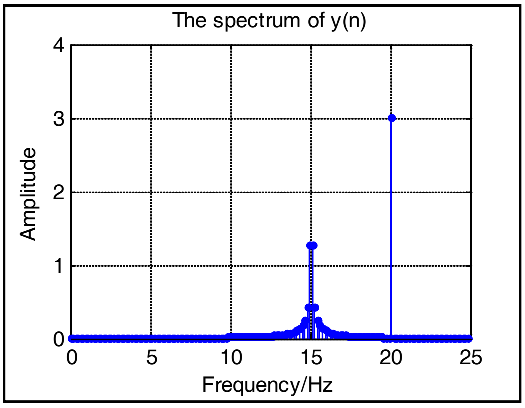

Fig. 4: The spectrum of y(n)

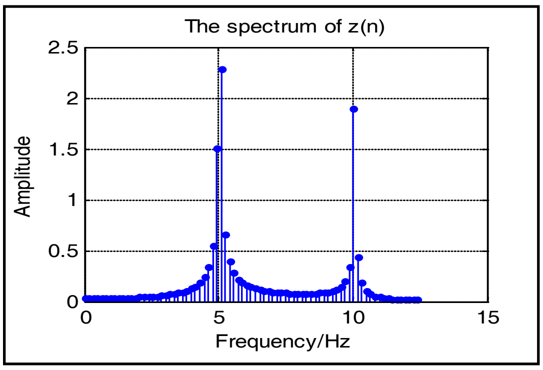

Fig. 5: The spectrum of z(n)

Sequence y(n)=x(2n) is the result of 2 times extraction of sequence x(n), as well as sequence z(n)=x(4n) is the result of 4 times extraction of sequence x(n). It could be gotten from figure 3 to 5 that the sampling frequency of 2 times extraction is 50 Hz, and it satisfy the sampling theorem, so the signal spectrum is right; As well as the sampling frequency of 4 times extraction is 25 Hz, and it does not satisfy the sampling theorem, so spectrum aliasing generated. Comparing of figure 3 and 4, it could be seen that the signal spectrum expand to 2 times of the original one after 2 times extraction.

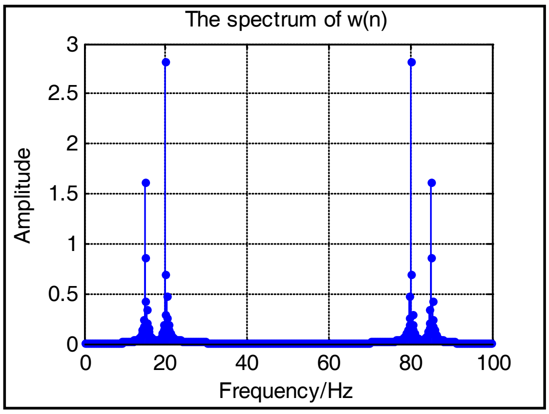

Fig. 6 : The spectrum of w(n)

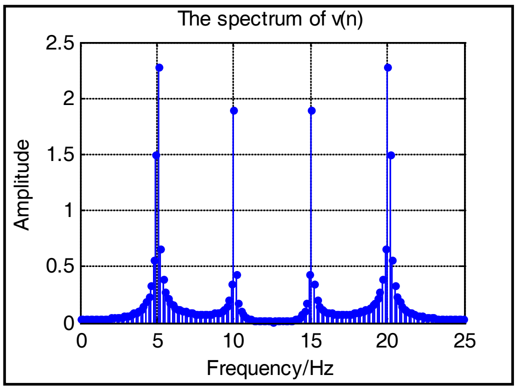

Fig. 7 : The spectrum of v(n)

Sequence w(n)=x(n/2) is the result of 2 times interpolation of sequence x(n), as well as sequence v(n)=z(n/2) is the result of 2 times interpolation of sequence z(n). Comparing of figure 3 and 6, it could be seen that the signal spectrum compressed to 0.5 times of the original one after 2 times interpolation. It could be seen from figure 7 that spectrum aliasing still exists after 2 times interpolation of signal z(n) which does not satisfy sampling theorem.

V. Conclusion

To avoid generating the phenomenon of spectrum aliasing, the sampling theorem must be satisfied in the process of integer times extraction, and the gotten signal spectrum expand to M times of the original one; While the signal spectrum compressed to 1/L times of the original one after L times interpolation.

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